

The flow properties of granular magnesia

T. M. JONES AND N. PILPEL

Measurements have been made of the flow of granular magnesium oxide through circular orifices and the equation

$$D_o = (1.6822 D_{P_{av}} + 1.9779) \left(\frac{4W}{60\pi \rho_P \sqrt{g}} \right)^{0.2571 - 0.0855 \log D_{P_{av}}}$$

has been derived. This is a particular form of a more general equation

$$D_o = A \left(\frac{4W}{60\pi \rho_P \sqrt{g}} \right)^{1/n}$$

which can be shown to apply to a wide range of materials, A and 1/n being functions of particle size and shape.

The equation has been employed for predicting the flow rates of binary and ternary mixtures of magnesia with an accuracy of 5%.

NUMEROUS equations have been published for the flow of powders through circular orifices. Some are purely empirical in nature (Bingham & Wikoff, 1931; Newton, Dunham & Simpson, 1945; Gregory, 1952; Franklin & Johanson, 1955; Luk'yanov, Gusev & Nikitina, 1960; Beverloo, Leniger & Van der Velde, 1961; Ciborowski & Badzynski, 1963); others have been derived from dimensional analysis (Deming & Mehring, 1929; Takahashi, 1933; Rausch, 1949; Brown & Richards, 1959; Fowler & Glastonbury, 1959; Rose & Tanaka, 1959) and others from consideration of the energy of a moving column of powder (Brown, 1961a; Harmens, 1963).

In most cases the equations have been derived from data pertaining to a large number of different materials, e.g. glass spheres, sand, coal dust and cereal grains, this being necessary to cover a reasonable range of particle sizes. As a result, it has been necessary to include in the equations parameters such as particle shape, density and surface roughness. But with the availability now of granular materials whose physical properties remain appreciably constant over a wide range of sizes, it has become possible to eliminate the effects of some of these variables. In principle this should enable a more accurate flow equation for a particular material to be formulated than heretofore.

The majority of the equations currently available apply primarily to systems containing monodispersed particles. But real powders invariably consist of mixtures of particles. From the purely practical point of view it would be advantageous to have a flow equation that could be equally applied to mono, binary and higher systems of a particular material.

In the present investigation, therefore, a study has been made of the flow behaviour of both single and multicomponent systems of magnesia. One objective has been to derive an equation which can be used to predict the flow rate of any mixture whose composition is known. At the same time it has been found possible to explain certain apparent discrepancies between several of the previously published flow equations.

From the School of Pharmacy, Chelsea College of Science and Technology, Manresa Road, London, S.W.3.

Experimental

PREPARATION OF MATERIALS

A batch of a hard grade of granular magnesia from the Washington Chemical Company containing 1.4% by weight of material volatile at 150° and 5.7% volatile at 800°, was sieved into 13 fractions in the size range 0.0032 to 0.2812 cm using British Standard sieves. Immediately before use, the fractions were resieved for 3 min with the Alpine Airjet sieve, using 20 g quantities, and were stored in tightly stoppered glass vessels. Some of the physical properties of the fractions are presented in Table 1, particle densities having been measured by immersion in toluene at 25° (Bauer, 1949) and bulk and tap densities by the British Standard (1948) method. Microscopic examination of particles taken from each of the sieve fractions revealed that, for practical purposes, they were all approximately spherical in shape and of similar rugosities.

TABLE 1. PROPERTIES OF DIFFERENT SIEVE FRACTIONS

B.S.S. size	Arithmetic mean sieve diameter (cm)	Density (g/ml)		
		Particle	Bulk	Tap
6/8	0.2435	3.490	0.910	0.996
8/10	0.1866	3.456	0.856	0.933
10/16	0.1340	3.460	0.860	0.941
16/22	0.0851	3.445	0.860	0.930
22/36	0.0561	3.458	0.870	0.930
36/52	0.0358	3.456	0.856	0.938
52/72	0.0253	3.458	0.887	0.970
72/150	0.0158	3.431	0.903	0.985
150/200	0.0090	3.458	0.920	0.988
200/240	0.0071	3.465	0.937	1.032
240/300	0.0059	3.443	0.994	1.063
300/350	0.0048	3.439	1.000	1.095
350/DIN 4188 32 μ	0.0038	3.401	1.039	1.127

Binary and ternary mixtures of the various sieve fractions were made up by weight, employing a total of 400 g of powder. Equal amounts of the components were mixed together and the remaining material was then added incrementally, the mixing itself being effected by pouring the powder 30 times from one 1 litre beaker into another.

APPARATUS

The apparatus for the flow rate measurements consisted of a vertical copper tube, internal diameter D_c of 3.82 cm, fitted with a base plate of perspex. This held a shutter and a sliding aluminium sheet, $\frac{1}{16}$ inch thick, into which had been accurately cut with a lathe, six circular orifices with mean diameters D_o of 0.603, 0.740, 0.898, 1.140, 1.353 and 1.686 cm (see Fig. 1).

PROCEDURE

The tube was filled with 400 g of powder. For mixtures the method of filling was to introduce 20 g of the mixture; all but 20 g was then poured into the second beaker and the final 20 g was then introduced into the tube. The rejected 360 g was mixed three times by pouring

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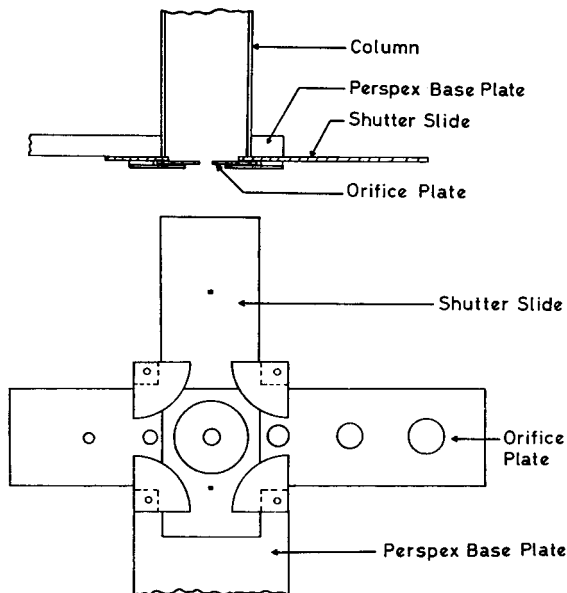


FIG. 1. Upper diagram: side elevation of flow rate apparatus. Lower diagram: plan of orifice/shutter arrangement.

between the two beakers and the process was repeated until the tube was full. Sieve analysis on each 20 g fraction before its introduction into the tube showed that the maximum variation in the compositions of mixtures throughout the length of the tube was $\pm 9\%$ (w/w) the average variation was $\pm 4.5\%$ (w/w), and this was considered satisfactory.

Flow rates, W g/min, were determined in triplicate by collecting and weighing the powder that escaped from the tube in fixed time intervals ranging from 5 to 60 sec. Shutter manipulation and incipient failure of the powder column (Jenike, 1962) were found to affect the flow rate in the initial stages. Also, when the head of powder fell below $2 \times D_c$, the flow rate increased. For these reasons the measurements were restricted to the (approximate) central one half of the powder column, when the maximum difference between separate determinations was found to be 5%.

Measurements were made on 13 mono, about 50 binary and about 30 ternary systems.

Results

Fig. 2 shows the effect of particle size and orifice diameter on the flow rates of monodispersed sieve fractions. Provided the orifice was greater than $6 \times D_p$ (Langmaid & Rose, 1957) maximum flow occurred when the particles were 0.0253 cm in diameter.

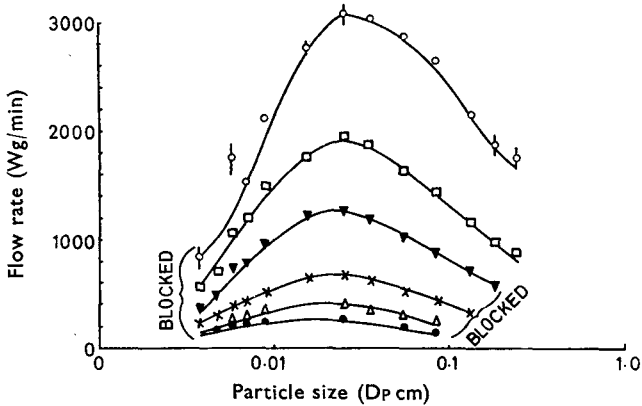


FIG. 2. Effect of particle size on flow of monodisperse systems. Orifice diameters (D_0) in cm: ○, 1.686; □, 1.353; ▼, 1.140; ×, 0.898; △, 0.740; ●, 0.603.

Fig. 3a is representative of the effect that orifice diameter had on the flow rate of a particular sized powder (>0.0253 cm) when a second component, also >0.0253 cm, was added in various amounts.

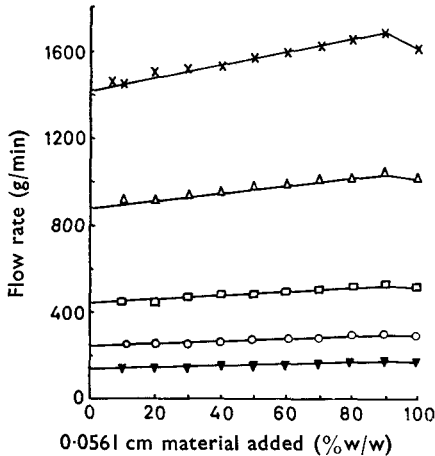


FIG. 3A. Effect of orifice diameter and composition on flow rate of binary mixtures of 0.0851 and 0.0561 cm particles. Orifice diameter in cm: ×, 1.353; △, 1.140; □, 0.898; ○, 0.740; ▼, 0.603.

Fig. 3b is representative of the effects produced by changing the size of this second component while maintaining the orifice constant.

It is seen that the addition of a smaller sized powder to a larger one increased its flow rate up to a maximum value at about 90% addition.

The flow behaviour of typical ternary systems, in which all three components were larger than 0.0253 cm, are shown in Figs 4a and 4b. The contour lines enclose the systems having similar flow rates. It is

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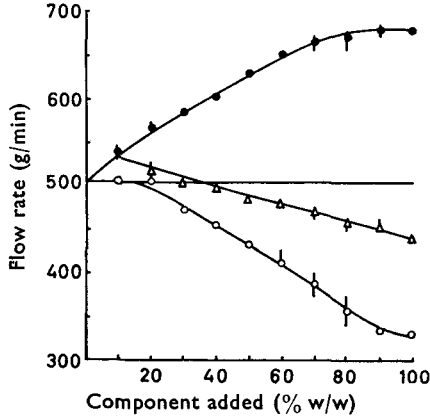


FIG. 3B. Effect of composition and size of additive on flow rate of 0.0561 cm particles in binary mixtures. Orifice diameter 0.898 cm. Additive size in cm: ●, 0.0253; △, 0.0851; ○, 0.1340.

seen that the addition of a small sized third component increases the flow rates of mixtures of two larger sizes and that, as with the mono and binary systems, the larger the value of D_0 , the higher the flow rate.

χ^2 tests on the reproducibility of the results were, in all instances, in excess of 0.99 probability.

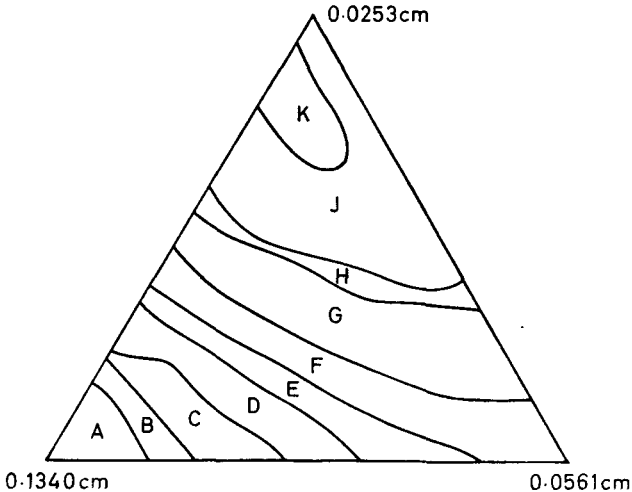


FIG. 4A. Flow rate of ternary mixtures. Orifice diameter 1.353 cm. A, 1100–1200. B, 1200–1300. C, 1300–1400. D, 1400–1500. E, 1500–1600. F, 1600–1700. G, 1700–1800. H, 1800–1900. J, 1900–2000. K, >2000. (g/min).

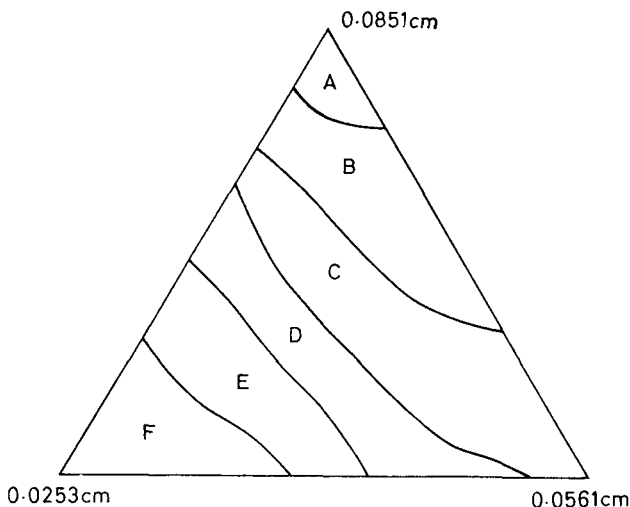


FIG. 4B. Flow rate of ternary mixtures. Orifice diameter 0.740 cm. A, <250. B, 250-280. C, 280-300. D, 300-350. E, 350-380. F, 380-400. (g/min).

Discussion

MONO SYSTEMS

The shapes of the curves in Fig. 2 are similar to those that have been obtained on other materials (Fowler & Glastonbury, 1959; Rose & Tanaka, 1959) though as far as can be ascertained, this is the first occasion on which particles of one material having a constant shape, density, surface roughness have been investigated over such a wide range of sizes.

When the particles are less than 0.0253 cm, complications in the flow arise due to mechanical factors and the operation of van der Waals', electrostatic and surface tension forces between particles (Pilpel, 1964). The present analysis has therefore been restricted to particles >0.02 cm where there is a steady decrease in flow rate with increasing particle size.

Some of the flow equations that have been proposed in the past have included a term for the bulk density, ρ_B , of the powder (Fowler & Glastonbury, 1959). But in more recent work this has been replaced by the particle density ρ_P , since at sizes greater than 0.02 cm the flow rate is independent of the tightness of the initial packing, i.e. independent of ρ_B (Brown & Richards, 1959; Harmens, 1963). It has been shown (Brown & Richards, 1960), for example, that in many monodispersed systems in which $D_P > 0.0253$ cm, plots of $\left(\frac{4W}{60\pi\rho_P\sqrt{g}}\right)^{0.4}$ versus D_0 yield straight lines, whose intercepts, k , on the abscissa are some function of the empty annulus at the periphery of the orifice.

Expressing this in the form

$$4W \left(\frac{4W}{60\pi\rho_P\sqrt{g}}\right)^{0.4} = m D_0 + C \quad \dots \quad (1)$$

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where C is an extrapolated negative intercept on the ordinate, it follows that

$$D_o = \frac{1}{m} \left(\frac{4W}{60\pi \rho_p \sqrt{g}} \right)^{0.4} - \frac{C}{m} \quad \dots \quad (2)$$

Now clearly $-\frac{C}{m} \equiv k$

thus
$$D_o \equiv \frac{1}{m} \left(\frac{4W}{60\pi \rho_p \sqrt{g}} \right)^{0.4} + k \quad \dots \quad (3)$$

and this is a general equation which applies to such varied materials as coal, glass beads, sand and tapioca (Brown & Richards, 1960).

Its applicability to the present experimental results on the mono systems of magnesia has been tested in Fig. 5, when good straight lines are obtained for the various values of D_p . The slopes, which have been obtained by regression analysis, have been plotted against particle size and obey the relationship

$$1/m = 0.5244 D_p + 1.9738 \quad \dots \quad (4)$$

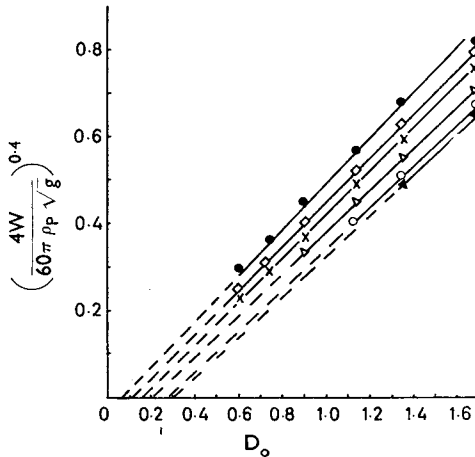


FIG. 5. Plot of $\left(\frac{4W}{60\pi \rho_p \sqrt{g}} \right)^{0.4}$ against orifice diameter D_o cm. Particle size in cm: ●, 0.0253; □, 0.0561; ×, 0.0851; △, 0.1340; ○, 0.1866; ▼, 0.2435.

It had previously been shown (Brown, 1961b) that for the materials already mentioned, k is qualitatively related to D_p ; this can be seen in a limited way from the data in Table 2. But with the present grade of magnesia in which the densities, shapes and rugosities of the particles are similar, a quantitative relationship between k and D_p may be expected. Log k has been plotted versus log D_p when a good straight line graph is obtained in which the maximum deviation is $\pm 3\%$. From this

$$k = 0.8375 D_p^{0.7} \quad \dots \quad (5)$$

TABLE 2. VARIATION OF k WITH PARTICLE SIZE

Material	Size (cm)	k	Reference
Magnesia	0.0158	0.005	Present work
	0.0253	0.065	
	0.0358	0.07	
	0.0561	0.11	
	0.0851	0.15	
	0.1340	0.19	
	0.1866	0.28	
Sand	0.0170	0.036	Brown 1961(a)
	0.0380	0.079	
	0.0540	0.12	
Glass beads ..	0.0230	0.041	
	0.0960	0.13	
Rounded sand ..	0.0620	0.10	
Coal	0.0610	0.14	
Tapioca	0.1450	0.22	

Now we can eliminate k, which is a function of D_p from equation (3) by writing it in the form

$$D_o = A \left(\frac{4W}{60\pi \rho_p \sqrt{g}} \right)^{1/N} \dots \dots \dots (6)$$

where $1/N$ and A are also functions of D_p . This in turn can be written

$$D_o \propto \left(\frac{4W}{60\pi \rho_p \sqrt{g}} \right)^{1/N} \dots \dots \dots (7)$$

and it is seen that the expression (7) is formally similar to the well known relationship

$$D_o \propto W^{1/n} \dots \dots \dots (8)$$

which has been shown to apply to a very wide range of different materials (references already cited). It may be inferred, therefore, that the exponents $1/n$ and $1/N$ should, in fact, be identical.

Many workers, using a wide variety of materials (Ketchum, 1911; Franklin & Johanson, 1955; Beverloo & others, 1961), have reported n as lying between $1/2.5$ and $1/3.1$. It is implicit in their data that a functional relationship should exist between $1/n$ and D_p . But to establish this and thereby explain the several anomalous results that have been reported by Beverloo & others (1961) it is necessary to eliminate the complicating effects of particle density, shape and rugosity. This has been done with the present grade of magnesia, and we now find that from a plot of $1/n$ against $\log D_p$

$$1/n = 0.2571 - 0.0855 \log D_p \dots \dots \dots (9)$$

which is a linear relationship. (Clearly, having eliminated the effect of ρ_p , it is not now pertinent to use the present data for testing the validity of other equations in which ρ_p appears as a parameter.) The exponent, $1/n$, varies from $1/2.0$ at 0.0038 cm to $1/3.2$ at 0.2435 cm. The mean value of this range is $1/2.6$ and this approximates to the value of 0.4 obtained by dimensional analysis (Deming & Mehrling, 1929; Takahashi,

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1933; Rose & Tanaka, 1959; Beverloo & others, 1961), the same value, in fact, that appears in equation (3). Furthermore, when $10 > D_P > 1$ cm, $1/n$ is approximately $1/3$, which leads to the relationship $W \propto D_0^3$ originally established by Ketchum (1911).

To make use of equation (6) which, as we have seen, is simply another form of equation (3), we employ equation (9) to obtain $1/n$ ($\equiv 1/N$). Then plotting $\left(\frac{4W}{60\pi \rho_P \sqrt{g}}\right)^{1/n}$ versus D_0 for each particle size, should yield a series of straight lines passing through the origin. The result is shown in Fig. 6 and satisfactorily agrees with expectation. The slopes, A , of the lines have been obtained, as before, by regressional analysis; when plotted against D_P they yield the relationship

$$A = 1.6822 D_P + 1.9779 \quad \dots \quad \dots \quad \dots \quad \dots \quad (10)$$

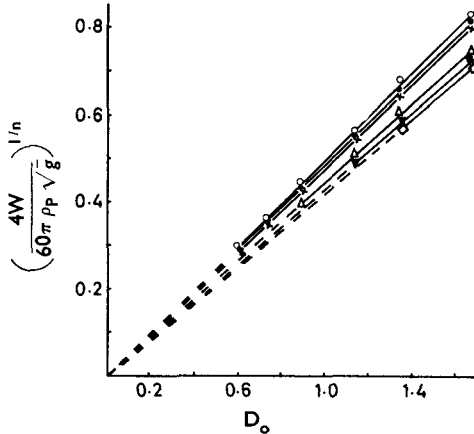


FIG. 6. Plot of $\left(\frac{4W}{60\pi \rho_P \sqrt{g}}\right)^{1/n}$ against orifice diameter D_0 cm. D_P in cm: \circ , 0.0253; \bullet , 0.0561; \times , 0.0851; \triangle , 0.1340; \blacktriangledown , 0.1866; \square , 0.2435.

Hence the particular form of equation (6) that is applicable to the grade of magnesia now employed is

$$D_0 = (1.6822 D_P + 1.9779) \left(\frac{4W}{60\pi \rho_P \sqrt{g}}\right)^{0.2571 - 0.0855 \log D_P} \quad \dots \quad (11)$$

There will, however, be occasions when it may be preferable to use equation (6) in its alternative form of equation (3), since this includes the empty annulus term k and this has physical significance. Substituting for $1/m$ and k from equations (4) and (5) into equation (3) we obtain, for magnesia

$$D_0 = (0.5244 D_P + 1.9738) \left(\frac{4W}{60\pi \rho_P \sqrt{g}}\right)^{0.4} + 0.8375 D_P^{0.7} \quad \dots \quad (12)$$

which gives equivalent results to equation (11).

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These equations have been used to predict the flow rates of the mono-dispersed systems and the results are given in Table 3. It is seen that the mean error is $\pm 3\%$, the maximum error is $\pm 4.2\%$ and this is considered very satisfactory.

TABLE 3. FLOW RATES OF MONODISPERSED SYSTEMS

Particle size (cm)	D ₀ (cm)	W _{calc} (g/min)	W _{obs} (g/min)	Error
0.0253	0.74	398	397-399	0%
	1.353	1842	1923-1945	-4.2%
0.0561	0.74	302	291-293	+3%
	0.898	513	514-517	-0.13%
0.0851	0.898	432	435-437	-1%
	1.353	1400	1401-1413	-0.35%
0.1340	0.898	341	322-338	+0.9%
	1.353	1173	1128-1158	+2.5%
0.1866	1.353	980	936-953	+3.5%
	1.686	1952	1788-1942	+0.5%

The fact that both k and the exponent $1/n$ ($\equiv 1/N$) are functions of the particle size shows that it is inadvisable to employ equations which include only average values of k or of $1/N$ (Leva, 1959; Beverloo & others, 1961; Zenz, 1962), for predicting flow rates over a wide range of particle sizes. Clearly the same treatment which has been employed in the present work for obtaining equations (4), (5), (9) and (10) in the particular case of magnesia, should also yield the values of k , $1/m$, $1/n$ and A that have to be substituted into equations (3) or (6) for predicting the flow rates of other materials.

MIXTURES

So far we have been concerned only with the flow behaviour of mono sieve fractions. But by means of a simple procedure equation (11) can also be employed to predict the flow behaviour of the binary and of the ternary systems. The quantity D_p in equations (9), (10) and (11) is replaced by a geometrical mean diameter $D_{p_{av}}$. Being a geometrical mean, weighting is automatically given to the smaller sized particles which are known to influence flow rates most.*

For a binary system

$$D_{p_{av}} = \sqrt{D_{P_1}^{M_1/0.5} \times D_{P_2}^{M_2/0.5}} \quad \dots \quad (9)$$

For a ternary system

$$D_{p_{av}} = \sqrt[3]{D_{P_1}^{M_1/0.33} \times D_{P_2}^{M_2/0.33} \times D_{P_3}^{M_3/0.33}} \quad \dots \quad (10)$$

* While an arithmetic mean is normally used to characterise real powders where the particle size distribution is a continuous function, in the present mixtures, where the histogram is discontinuous, it is unrealistic to employ an arithmetic scale, e.g. a modal, median, mean weight or mean surface diameter. A χ^2 test showed a much closer fit between observed and predicted values of flow rate using a geometric mean than could be obtained with any conventional arithmetic mean.

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where M_1 is the mass fraction of particles of diameter D_{P_1}

$$\begin{aligned}
 M_2 & \text{ ,, ,, ,, ,, ,, ,, ,, } D_{P_2} \\
 M_3 & \text{ ,, ,, ,, ,, ,, ,, ,, } D_{P_3}
 \end{aligned}$$

$D_{P_{AV}}$ for quaternary and higher systems will be obtained similarly.

Values of $D_{P_{AV}}$ have been calculated for several of the binary and ternary systems employed in the present investigation and then substituted into equation (11). Tables 4 and 5 show that the resulting calculated values of flow rate W_{calc} , agree with the observed values on average to within $\pm 5\%$ and at worst to within $\pm 13\%$.

TABLE 4. FLOW RATES OF BINARY MIXTURES

Mixture				D_o (cm)	W_{calc} (g/min)	W_{obs} (g/min)	Error
D_{P_1}	Conc (% w/w)	D_{P_2}	Conc (% w/w)				
0-0561	20	0-1340	80	0-898 1-353	377 1267	341-372 1242-1290	+1-3% 0%
	50		50	0-898 1-353	430 1397	431-435 1405-1429	+0-025% -0-6%
	80		20	0-898 1-353	481 1512	501-510 1566-1607	-4% -3-4%
0-0561	20	0-0851	80	0-898 1-353	450 1444	448-462 1490-1510	0% -3%
	50		50	0-898 1-353	474 1498	482-489 1553-1591	-1-2% -3-5%
	80		20	0-898 1-353	497 1550	515-529 1645-1675	-3-5% -5-8%
0-0561	20	0-0253	80	0-74 1-14 1-353	379 1155 1797	391-396 1215-1233 1931-1941	-2-8% -4-9% -6-9%
	50		50	0-74 1-353	350 1724	361-365 1879-1900	-3-0% -8-2%
	80		20	0-603 1-353	185 1645	185-197 1725-1740	0% -5-6%
0-1340	80	0-0253	20	0-898 1-353	409 1348	324-352 1254-1274	+13-9% +5-6%
	50		50	0-898 1-353	507 1569	489-533 1689-1740	0% -7-1%
	20		80	1-353	1745	1932-2105	-9-7%
0-0851	20	0-0253	80	0-603 1-353	217 1772	238-242 2016-2039	-8-8% -12-1%
	40		60	0-74 1-14	340 1074	356-367 1180-1200	-4-5% -8-9%
	80		20	0-898 1-353	481 1514	466-482 1505-1548	0% -2-19%

By employing the procedure in reverse, it is possible to calculate the composition of any binary mixture to within about $\pm 12\%$ if its flow rate is measured and the sizes of the constituent particles are known. Both procedures might be useful in certain circumstances for quality control of magnesia in industry.

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TABLE 5. FLOW RATES OF TERNARY MIXTURES

Mixture (% w/w)			D _o (cm)	W _{calc} (g/min)	W _{obs} (g/min)	Error
D _p						
0.1340	0.0253	0.0561				
20	40	40	0.74	320	322-340	-0.6%
			1.14	1029	1144-1174	-10%
60	20	20	0.898	444	378-425	+4.5%
			1.14	877	844-910	0%
D _p			D _o (cm)	W _{calc} (g/min)	W _{obs} (g/min)	Error
0.0851	0.0561	0.0253				
20	40	40	0.898	556	591-608	-9.3%
			1.14	1054	1122-1188	-6.1%
50	10	40	0.603	181	179-187	0%
			1.14	978	1076-1102	-5.2%

CONCLUSION

The principle involved in the derivation of equations (11) or (12), namely the employment of an exponent $1/n$ and an annulus term k , both of which are functions of particle size, should be of general application to other granular materials.

k , $1/n$, A and $1/m$ are obtained as functions of particle shape and size, it should become possible to predict the flow rates of other sizes of the same materials through any sized circular orifice.

The equations that have been developed for granular magnesia in the present work provide, for the first time, a means of predicting its flow rate when particles of many different sizes are present.

NOTATION

A	An empirical function of D_p : equation (10).	k	A function of the empty annulus around the orifice (cm) dependent on D_p : equation (5).
D _c	Tube diameter (cm).	1/m	An empirical function of D_p : equation (4).
D _o	Orifice diameter (cm).	M	The mass fraction of particles at size D_p .
D _p	Mean arithmetic sieve diameter (cm).	1/n	An empirical function of D_p : equation (9).
D _{pav}	Geometric mean diameter of particles in a mixture (cm).	ρ _p	Particle density (g/ml).
g	Acceleration due to gravity (cm/sec ²).	ρ _B	Bulk density (g/ml).
		W	Flow rate (g/min).

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